Technical Comments

Comments on "Motion of Re-Entry Vehicles During Constant-Altitude Glide"

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IN a recent technical note, Wang presented some interesting results concerning a closed-form solution for the motion of a re-entry vehicle during constant-altitude glide. The only assumption in his solution is that $\sin\phi = 1$, where ϕ is defined as the roll angle of the vehicle measured from the local vertical. It is this author's opinion that this assumption is much more severe than that of neglecting the "lateral centrifugal force" term in the complete equations of motion. A closed-form solution of the yaw equation, neglecting the "lateral centrifugal force" term and letting the roll angle vary in the proper manner, is presented here.

Using Wang's notation, Eq. (7) of Ref. 1 under the foregoing assumption becomes

$$V(d\psi/dV) = -(L/D)\sin\phi \tag{1}$$

From Eq. (2) of Ref. 1,

$$\sin\phi = \{1 - C^2[(1/\bar{V}^2) - 1]^2\}^{1/2} \tag{2}$$

Expanding Eq. (2) and substituting into Eq. (1), the integral to be evaluated is

$$I = \int_1^{\phi=0} \sin\phi\,\frac{dV}{V} = -\frac{1}{2}\int X^{1/2}\frac{dx}{x} \tag{3}$$

where

$$X = \sin^2\phi$$
$$= (1 - C^2) + 2Cx - x^2$$
$$x = C/\bar{V}^2$$

The solution of this integral is

$$-2I = X^{1/2} - C\sin^{-1}(C - x) + (1 - C^2)F(1 - C^2) \tag{4}$$

The function F may be expressed in the following forms:

$$F(1 - C^2) = \frac{-1}{[1 - C^2]^{1/2}} \times \log\left[\frac{X^{1/2} + (1 - C^2)^{1/2}}{x} + \frac{C}{(1 - C^2)^{1/2}}\right] \qquad C^2 < 1 \tag{5}$$

$$F(1 - C^2) = -(X^{1/2}/Cx) \qquad C^2 = 1 \tag{6}$$

$$F(1 - C^2) = \frac{1}{(C^2 - 1)^{1/2}}\sin^{-1}\left[\frac{Cx + (1 - C^2)}{x}\right] \qquad C^2 > 1 \tag{7}$$

At the equilibrium glide boundary ($\phi = 0$), we have

$$x = C + 1 \tag{8}$$

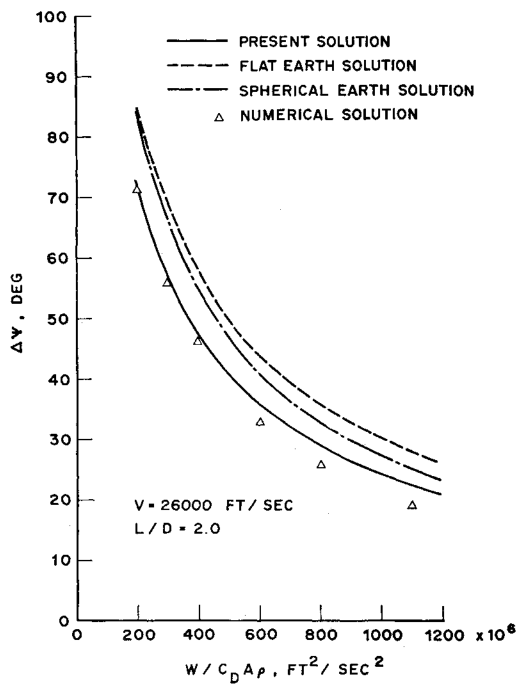

Fig. 1 Comparison of solutions.

The final value of the integral may be written as

$$-2I_2(\phi = 0) = C(\pi/2) + [1 - C^2]^{1/2}\log[(1 - C^2)^{1/2}] \qquad C^2 < 1 \tag{9}$$

$$-2I_2(\phi = 0) = \pi/2 \qquad C^2 = 1 \tag{10}$$

$$-2I_2(\phi = 0) = (\pi/2)[C - (C^2 - 1)^{1/2}] \qquad C^2 > 1 \tag{11}$$

The change in vehicle heading may now be expressed as

$$\Delta\psi = (L/D)[I_1 - I_2] \tag{12}$$

Figure 1 presents a comparison of some numerical data and

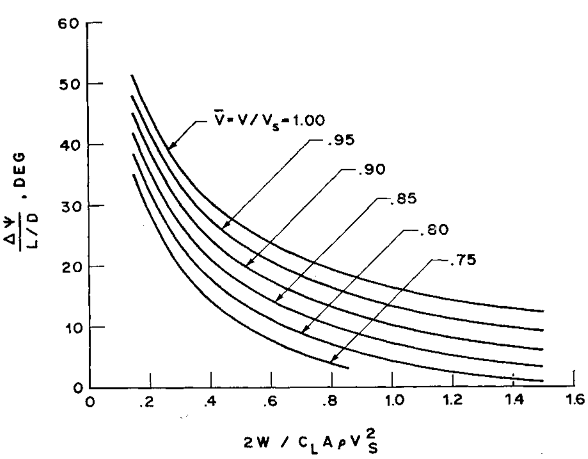

Fig. 2 Variation of $\Delta\psi/(L/D)$ as a function of $\bar{V}$ and $2W/(C_L A\rho\ V_s^2)$.

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the three closed-form solutions for the heading or yaw angle. It is noted that the solution just discussed agrees better with the numerical data than does the "spherical earth" solution. At large values of $W/C_DA\rho$, the preceding solution is better by approximately 50%, whereas at small values of $W/C_DA\rho$, the difference between the foregoing solution and the numerical data is extremely small. In view of the solution contained herein, it is apparent that a better closed-form solution for the heading angle may be obtained by omitting the "lateral centrifugal force" term and including the variation of the roll angle.

An interesting feature of the preceding solution is illustrated in Fig. 2. Knowing the vehicle characteristics, L/D and W/C_LA, and the velocity and altitude at which the constant-altitude flight regime begins, the change in the heading angle is available immediately.

Reference

[1] Wang, H. E., "Motion of re-entry vehicles during constant-altitude glide," AIAA J. **3**, 1346–1348 (1965).

Reply by Author to R. T. Theobald

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In the course of our investigation of the lateral motion of re-entry vehicles at constant altitudes, assumptions other than $\sin\phi = 1$ have been examined. One assumption examined was precisely the one presented by Theobald in the preceding comment. Another one was a constant $\sin\phi$ different from unity, and a third one was a piecewise solution involving a series of values of $\sin\phi$. These approaches have not been pursued because they do not produce simple solutions for lateral range and downrange. It is true that the turn angle solution can be made extremely accurate, but it is very difficult sometimes to go beyond that to obtain useful expressions for lateral range and downrange.

The $\sin\phi = 1$ approximation used in Ref. 1 is quite a simplification to the physical behavior of the vehicle motion. Surprisingly enough, however, it is mathematically sufficiently accurate to describe the motion of the vehicle. Assuming $\sin\phi = 1$ introduces excessive lateral turning. On the other hand, the total distance traveled by the vehicle between a given initial velocity and the equilibrium glide velocity is not at all affected by this approximation. Consequently, the approximate solution (assuming $\sin\phi = 1$) demands only a redistribution of the distance traveled between lateral range and downrange. Because of the excessive lateral turning, the approximate solution always produces higher lateral range and lower downrange. The inaccuracy in lateral range and downrange, however, is much less than that in the turning angle.

The main values of Ref. 1 are that the solutions permit quick estimates of ranges for re-entry analysis, and they bring about the general trends of the various parameters involved. In this sense, a parallel can be drawn between the $\sin\phi = 1$ approximation in this problem and the straight-line trajectory approximation for ballistic re-entry.

Reference

[1] Wang, H. E., "Motion of re-entry vehicles during constant-altitude glide," AIAA J. **3**, 1346–1348 (1965).

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Comment on "Magnetohydrodynamic-Hypersonic Viscous and Inviscid Flow near the Stagnation Point of a Blunt Body"

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In a recent note,[1] Smith, Schwimmer, and Wu describe an extension to the magnetohydrodynamic, hypersonic, stagnation flow theory proposed by Bush.[2] In principle, their development involves including the viscous term in the constant property analysis and solving the two-point boundary value problem from both ends at once rather than starting at the shock and marching to the body. As a consequence of experimental and analytical studies in our laboratory, we wish to make several comments regarding this contribution.

Concerning the viscosity, the authors of Ref. 1 have assumed a thin viscous boundary layer, so that although included in the differential equations, viscosity is neglected in their shock vorticity jump boundary condition (from the curl of the momentum equation evaluated behind the shock using Rankins-Hugoniot conditions). This same assumption was made in earlier work[3] of the authors. The assumption is clearly valid if the shock layer Reynolds number is high. In the numerical examples of Ref. 1, $Re = v_\infty r_b/\nu = 100, 1000, \infty$. The corresponding shock layer Reynolds numbers are $Re_1 = \epsilon v_\infty r_b/\nu = 10, 100, \infty$ for $\epsilon = \frac{1}{10}$. Validity of the assumption at very low Re_1 requires detailed consideration.

First of all, from a qualitative viewpoint, it is necessary to recognize that when the Reynolds number is very low, the shock layer and the shock will be merged fully. The Hugoniot relations are then no longer valid and the full-fledged Navier-Stokes equations must be satisfied from the front of the shock to the body with the compressibility included in the shock region.

A more detailed evaluation concerns the extent of the boundary layer within the shock layer. We assume that the boundary-layer thickness δ is given by Hiemenz stagnation point flow.[4] Thus,

$$\delta \approx 2.4\, (\nu/a)^{1/2} \tag{1}$$

where ν is kinematic viscosity and a is a constant determined by the flow outside of the boundary layer. If we match this flow to the Hugoniot shock downstream pressure, we obtain

$$\delta/\Delta \approx 2.4\, Re_\Delta^{-1/2} \tag{2}$$

where Δ is the shock standoff distance and Re_Δ the shock layer Reynolds number based on Δ. If we denote by Re_1, the corresponding Reynolds number based on body radius r_b, we get

$$\delta/\Delta \approx 2.4[(\Delta/r_b)Re_1]^{-1/2} \tag{3}$$

For a shock density ratio of $\epsilon = \frac{1}{10}$ and $Re_1 = 100$, it is shown in Ref. 1 that $\Delta/r_b \approx 0.1$. Hence

$$\delta/\Delta \approx 2.4/3.16 = 0.76$$

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